A STOCHASTIC APPROACH TO ANALYZE STUDENTS' PERFORMANCE IN HIGHER EDUCATION

Elmabrok H. Abdelrahim
Faculty of Engineering
University of Benghazi, Libya
elmabrokabdelrahim@gmail.com

Abstract

In today's competitive society, the needs to evaluate the quality of the educational organizations outputs is vital whether it was governmental or private. Also, being able to evaluate students' progress is useful for any educational system. It gives a better understanding of how students resonates and it can be used as support for important decisions and planning. In this article, an application of absorbing Markov chain in the analysis of students' performance at one of the engineering faculty departments at Benghazi university. The study programme was modeled by a finite Markov chain with eight transient and two absorbing states. The probability transition matrix was constructed and the students' progression towards the next stage of the study programme was estimated. The expected time that a student spends at a particular stage as well as the expected duration of the study is determined. From any particular stage, the estimated fraction of students which will be dismissed or graduate can be also obtained.

Key words: Markov Chain, Higher education, Students' performance

الملخص

في مجتمع اليوم التنافسي أصبحت الحاجة إلى تقييم جودة مخرجات المؤسسات التعليمية سواء كانت حكومية أو خاصة أمراً حيوياً، كما أن القدرة على تقييم تقدم الطلاب أمر مفيد ومهم لأي نظام تعليمي. كذلك القدرة على هذا التقييم توفر فهمًا أفضل لكيفية تفاعل الطلاب مع النظام المتبع مما يجعله ركيزة اساسية لدعم القرارات والتخطيطات المهمة الهادفة لتطوير هذا النظام. يتناول هذا المقال تطبيق سلسلة ماركوف الممتصة في تحليل أداء الطلاب بأحد أقسام كلية الهندسة بجامعة بنغازي حيث تم نمذجة البرنامج الدراسى باستخدام سلسلة ماركوف المحدودة ذات ثماني حالات طاردة وحالتين ممتصتين ، وبناء عليه تم انشاء مصفوفة الانتقال الاحتمالية وتم تقدير تقدم الطلاب نحو المرحلة التالية من برنامج الدراسة. بالاظافة لما سبق، تم تقدير الوقت المتوقع الذي يقضيه الطالب في مرحلة معينة وكذلك المدة المتوقعة للدراسة. وأخيراً ، من أي مرحلة معينة من مراحل الدراسة يمكن الحصول على الجزء المقدر من الطلاب الذين سيتم فصلهم أو تخرجهم.

الكلمات المفتاحية: سلاسل ماركوف ، التعليم العالى ، أداء الطلاب

Introduction:

To create a better future in our life, the education must have the higher priority. The greatest concern in higher education these days are the student's academic performance and graduation rates. Undoubtedly, the educational planning needs a practical tool to evaluate the current process in order to improve it, for that, the creation and improving the mathematical models which will help in educational process assessment continued since many years ago. Because of the continuous increasing in the tertiary education the enrollment predicting is very important for the educational planning. According to the World Bank, the tertiary education was increased by 19% during 17 years (from 2000 to 2017) [Khairun and Husna, 2021].

The main property of a Markov chain is that, the estimated probabilities for the future events depend only on the current state of the process, and it is independent of the history of the process. Markov chain can be described as a mathematical model that simulating the dynamic behavior of special kinds of processes.

The main properties of Markov chain are:

for a process that occur during periodical time interval;

a) The possible outcomes are finite and called states.

b) The probability of moving from one state to another called transition probability and it is fixed.

c) The probability of moving from one state to another state (even to the same state) is depends only on the two states.

The states can be finite or infinite. When the states are finite the Markov process is a Markov chain. The main objective of this study is to monitor the students' performance and to observe their progress towards a prefixed goal utilizing the absorbing Markov chain. The Markov chain will be applied to evaluate students' performance in one of the departments in Engineering faculty at Benghazi university. The students' transition information within the six semesters which constitute the department program was collected from the registrar office for six consecutive semesters, starting from the academic semester of Spring 2014/15 to Spring 2018/19. The progress of students was tracked during this period of time considering the students' various changes in registering status. (i.e., withdrew, suspended, dismissed, and graduated). The students who did not pass the units that allow them to transferring to the next semester level (based on the number of units allocated by the department program for each semester) will be allocated at the same previous level. The number of semesters required to graduate from the department program are six in addition to the two semesters in the general sciences department before students are assigned to departments. Based on these data, the states of the process were defined. The data were then categorized according to the states and later transformed into the transition probability matrix (PTM).

LITERATURE REVIEW

The absorbing Markov chain model which introduced by Khairun and Husna (2021) demonstrated its ability to describing the students' enrollment behavior at School of Mathematical Sciences, University Sains Malaysia. The model was applied for the academic programs and gender to study the absorption, retention and repetitive rates of the students. The undergraduate student's enrollment from 2016/2017 until 2018/2019 were their targeted population. In edition to the PTM, the fundamental matrix (N) was determined as well to predict the expected duration of study until graduating and the enrollment rates is also estimated. Aparna and Sarat (2017) used the absorbing Markov chain model at Gauhati University for different six colleges. The proposed model was for analyzing the students' performance in order to evaluating the education system in that district. Almost of the performance measures in this study was same the introduced measures by the previous study and the different was in the targeted population patterns. An absorbing finite Markov chain model with seven states (five transient and two absorbing) introduced by Brezavšček, Bach and Baggia (2017). The aim was monitoring the quality and effectiveness indicators for Slovenian higher education institution programme. From 2008/09 until 2016/17 the data collected to construct the PTM and N. The expected enrolment for the three years next was determined as well as the duration of time until graduate or dismissed starting from any particular stage. Moreover, the period of time that will be spent at any stage and the fraction of students those will graduate or leave the programme were estimated. The using of absorbing Markov chain in analyzing the students' performance in higher education not limited to the previous introduced studies where more studies could be find such as introduced by Nyandwaki (2014), Adekele and Oguntuase (2014) , Amiens and Oisamoje (2016), Auwalu, Mohammed, and Saliu (2013) and Rahim R., et al., (2013) . In this article a finite absorbing Markov chain will be used to analyze the students' performance at a department in engineering faculty.

I. METHODOLOGY

A. Markov Chain

A Markov chain is a family of random variables $\{X_n \mid n \geq 0\}$ indexed by discrete-time where each X_n is an S-valued random variable for some finite state space S = {1, . . ., s} and satisfies the following Markov property: for all positive integers n, and for all choices of i0, . . ., in−2, i, j ϵ S,

$$\Pr(X_n = j|X_{n-1} = i, X_{n-2} = i_{n-2}, \ldots X_0 = i_0) = P_r(X_n = j|X_{n-1} = i \quad (1)$$

Equ. (1) demonstrates the main property of Markov chains that the present state is the only relevant information for the future. The initial probability distribution X_0 with the PTM P defines the probabilities for transitions from any status to another in the Markov chain [Kemeny and Snell, 1960].

Theorem: Let X_n be a Markov chain and S is the possible state space S = {1, . . ., s} and PTM P. Then for all $i, j \in$ S and positive integers n, m,

1. $\sum_{k=1}^{s} P_{ik} = 1$
2. $P_r(X_n = j|X_0 = i) = P_{ij}^n$
3. Chapman-Kolmogorov Equation:

$$P_{ij}^{m+n} = \sum_{k=1}^{s} P_{ik}^m P_{kj}^n$$

Corollary: Let $\alpha^{(n)}$ be the initial probability of the initial state X_0 of a Markov chain with PTM P. Then the probability that the chain is in state *i* after *n* steps is the *i*th entry of the vector [Kemeny and Snell, 1960]:

$$\alpha^{(n)} = \alpha^0 P^n \quad (2)$$

B. Absorbing Markov Chains

Definition: A state *i* of a Markov chain is called absorbing if it is impossible to leave it (P*ii* = 1). A Markov chain is absorbing if it has at least one absorbing state that is accessible from every other state, while, the non-absorbing states in the chain are transient. [Kemeny and Snell, 1960].
The canonical form of the PTM is:

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix} \quad (3)$$

Where,

- Q is $t \times t$ transitions probability between the transient states
- R is $t \times r$ transition probability from the transient states to the absorbing states
- is $r \times t$ zero matrix
- I is $r \times r$ identity matrix

- **Definition:** The matrix

$$N = (I - Q)^{-1} \tag{4}$$

is called the fundamental matrix (N) for P.

Where I denote the identity matrix of size $t \times t$. The n_{ij} in N shows the average number of times a Markov chain in the transient state *j* when it started in the transient state *i*.

The number of expected transitions until a Markov chain is absorbed into the absorbing states when it started in the transient state *i;* denoted as the expected time until absorption (μ_i). It can be calculated from:

$$\mu = N1 \tag{5}$$

Where 1 is the column identity vector [Brezavšček et al., 2017].

The probability to be in the absorption $\boldsymbol{f_{ij}}$ (in the absorbing state when starting from any transient state) can be obtained from the matrix ***f*** as the follows:

$$f = NR \tag{6}$$

Where ***R*** is the sub-matrix from the PTM P in (3).

At any given time n the distribution over states is a stochastic row vector and can be as follows:

$$P^{(n)} = P^{(0)}.P^{n} \tag{7}$$

$P^{(0)}$ is the initial vector (initial distribution) and each $p_i^{(n)}$ of $p^{(n)}$ denote the probabilities that the chain is in the state *i* in time *n* [Brezavšček et al., 2017].

In this work, the students' transition movement within the department programme will be modeled as an absorbing Markov chain. The chain contains eight transient statuses which the student can be in any one of them, and two absorbing status representing graduation or dismissed events.

In the rest of this article, the chain for the considered case study will be designed as well as the indicators that will be used to evaluate the performance of the students will be calculated.

II. THE CASE STUDY

The target department programme containing six academic semesters each of them representing a transient state. In addition to the previous six transient states, there are two transient states the student can be in one of them which are withdrawal (W) and freezing (F). finally, if the student not in one of the above eight transient states then, he must be graduated (G) or dismissed (D). Depending on the previous statuses the chain can be modeled as the following:

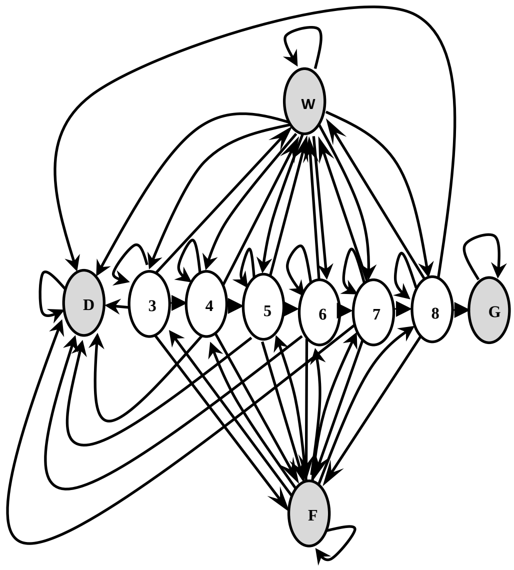

Fig.1 The department students' transition chain

According to the previous chain in fig. 1; the PTM can be written as shown in fig. 2.

A. Data Collected

To apply the previous indicated model (absorbing Markov chain) the data were collected from the students' records in the concerned department at faculty of engineering, university of Benghazi. The frequency data during six consecutive academic semesters from spring 2014/15 to spring 2018/19 are listed in Table 1.

P =

	3	4	5	6	7	8	f	W	D	G
3	P_{33}	P_{34}	0	0	0	0	P_{3f}	P_{3w}	0	0
4	0	P_{44}	P_{45}	0	0	0	P_{4f}	P_{4w}	P_{4D}	0
5	0	0	P_{55}	P_{56}	0	0	P_{5f}	P_{5w}	P_{5D}	0
6	0	0	0	P_{66}	P_{67}	0	P_{6f}	P_{6w}	P_{6D}	0
7	0	0	0	0	P_{77}	P_{78}	P_{7f}	P_{7w}	P_{7D}	0
8	0	0	0	0	0	P_{88}	P_{8f}	P_{8w}	P_{8D}	P_{8G}
f	P_{f3}	P_{f4}	P_{f5}	P_{f6}	P_{f7}	P_{f8}	P_{ff}	P_{fw}	0	0
W	P_{w3}	P_{w4}	P_{w5}	P_{w6}	P_{w7}	P_{w8}	P_{wf}	P_{ww}	0	0
D	0	0	0	0	0	0	0	0	1	0
G	0	0	0	0	0	0	0	0	0	1

Q ← ; → R

Fig. 2 : PTM for the chain in Fig. 1.

B. Construction of PTM

The frequency data from Table 1 were used to estimate the transition probabilities of the transition matrix. First, we calculated "partial" transition probability matrices for a particular academic semester separately. For example, "p_{33}" in the P_1 matrix is calculated by dividing the value "59" in the transition matrix on the sum of the row. We obtained the matrices in Fig. 2. The expected probability transition matrix $\mathbf{P}_{\mathbf{ave}}$ (Fig.3) calculated as the average values considering all six academic semesters. In calculating the expected transition probabilities, we took into consideration only these entries from Table 1 where the frequency data are actually available.

C. Expected progression

Directly from the PTM ($\mathbf{P}_{\mathbf{ave}}$) the progression between successive stages for the students can be obtained. Particularly useful are the possibilities of moving from the third semester to the fourth semester, from the fourth semester to the fifth semester and so forth, as well as from the eighth semester to graduation within one academic semester. These probabilities represent the fraction of students who progressed successfully during one semester. The results of our analysis are presented in Table 2.

D. Expected time spends by students at the given level and estimated study period

Using the sub-matrices Q1-Q6 from the probability transitions matrices P1-P6 the fundamental matrices N1-N6 were calculated. Also, the fundamental matrix N that correspond the expected probability transitions matrix $P_{\mathbf{ave}}$ is obtained. The elements of the fundamental matrices represent the expected number of academic semesters when the student is enrolled in a particular stage of the study. For example, let we assume an average student who is currently enrolled in the third semester. During his enrolment in the study program, it is expected that he will spend 2.404 academic semester for the third semester, 1.752 academic semester for the fourth semester, 1 academic semester for the fifth semester, 1 academic semester for the sixth semester, 1 academic semester for the seventh semester, 1 academic semester for the eighth semester, 0.263 academic semester he will be freeze, while 0.084 academic semester he will be withdraw.

The row sum of the fundamental matrix's entries represents the expected time until absorption from a given transient state and can be interpreted as the expected duration of the study starting at specific study stage (i.e., the expected enrolment in the study program until graduation or dismissal) (see Table 3). The sum of the diagonal elements of the fundamental matrix gives us the expected duration of the study from the third semester until graduation. The results are shown in Table 4

Table 1. Students' transitions through semesters 2014/15 TO 2018/1

Spring: 2014/2015

From \ To	3	4	5	6	7	8	F	W	D	G	Sum
3	59	31	0	0	0	0	1	0	0	0	91
4	0	15	28	0	0	0	0	0	4	0	47
5	0	0	1	20	0	0	0	1	2	0	24
6	0	0	0	11	21	0	0	0	0	0	32
7	0	0	0	0	11	19	0	0	3	0	33
8	0	0	0	0	0	7	0	0	0	21	28
F	10	2	3	2	1	1	2	0	0	0	21
W	6	5	5	3	4	2	0	0	0	0	25
New student	16	0	0	0	0	0	0	0	0	0	16

Spring: 2017/2018

From \ To	3	4	5	6	7	8	F	W	D	G	Sum
3	47	56	0	0	0	0	0	1	0	0	104
4	0	32	29	0	0	0	0	1	4	0	66
5	0	0	5	28	0	0	0	0	0	0	33
6	0	0	0	12	32	0	0	1	0	0	45
7	0	0	0	0	17	23	1	1	2	0	44
8	0	0	0	0	0	3	0	0	0	22	25
F	0	0	0	0	0	0	0	0	0	0	0
W	7	1	1	2	1	1	0	1	0	0	14
New student	10	0	0	0	0	0	0	0	0	0	10

Spring: 2016/2017

From \ To	3	4	5	6	7	8	F	W	D	G	Sum
3	42	26	0	0	0	0	21	2	0	0	91
4	0	19	27	0	0	0	5	1	1	0	53
5	0	0	10	21	0	0	6	0	0	0	37
6	0	0	0	12	19	0	5	0	0	0	36
7	0	0	0	0	7	24	4	0	2	0	37
8	0	0	0	0	0	11	2	0	0	16	29
F	27	2	2	3	0	0	0	0	0	0	34
W	8	1	1	3	0	2	0	0	0	0	15
New student	41	0	0	0	0	0	0	0	0	0	41

Fall: 2018/2019

From \ To	3	4	5	6	7	8	F	W	D	G	Sum
3	30	34	0	0	0	0	0	0	0	0	64
4	0	36	46	0	0	0	2	1	4	0	89
5	0	0	9	24	0	0	1	0	1	0	35
6	0	0	0	13	28	0	0	1	0	0	42
7	0	0	0	0	25	22	0	0	3	0	50
8	0	0	0	0	0	9	0	0	1	16	26
F	1	1	0	0	0	0	0	0	0	0	2
W	7	5	0	0	0	2	0	0	0	0	14
New student	44	0	0	0	0	0	0	0	0	0	44

Fall: 2017/2018

From \ To	3	4	5	6	7	8	F	W	D	G	Sum
3	67	49	0	0	0	0	1	1	0	0	118
4	0	12	31	0	0	0	1	0	4	0	48
5	0	0	1	36	0	0	0	1	2	0	40
6	0	0	0	6	33	0	0	0	0	0	39
7	0	0	0	0	10	14	0	0	2	0	26
8	0	0	0	0	0	4	0	0	0	33	37
F	0	1	0	1	1	3	0	0	0	0	6
W	7	4	1	2	0	4	0	0	0	0	18
New student	30	0	0	0	0	0	0	0	0	0	30

Spring: 2018/2019

From \ To	3	4	5	6	7	8	F	W	D	G	Sum
3	47	33	0	0	0	0	0	2	0	0	82
4	0	40	33	0	0	0	0	1	2	0	76
5	0	0	14	38	0	0	2	0	1	0	55
6	0	0	0	15	19	0	1	2	0	0	37
7	0	0	0	0	21	30	0	0	2	0	53
8	0	0	0	0	0	15	0	0	0	18	33
F	3	1	0	1	0	0	0	0	0	0	5
W	0	1	2	0	2	0	0	0	0	0	5
New student	24	0	0	0	0	0	0	0	0	0	24

P1 =

	3	4	5	6	7	8	F	W	D	G
3	0.65	0.34	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
4	0.00	0.32	0.60	0.00	0.00	0.00	0.00	0.00	0.09	0.00
5	0.00	0.00	0.04	0.83	0.00	0.00	0.00	0.04	0.08	0.00
6	0.00	0.00	0.00	0.34	0.66	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.33	0.58	0.00	0.00	0.09	0.00
8	0.00	0.00	0.00	0.00	0.00	0.25	0.00	0.00	0.00	0.75
F	0.48	0.10	0.14	0.10	0.05	0.05	0.10	0.00	0.00	0.00
W	0.24	0.20	0.20	0.12	0.16	0.08	0.00	0.00	0.00	0.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
G	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

P4 =

	3	4	5	6	7	8	F	W	D	G
3	0.45	0.54	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
4	0.00	0.48	0.44	0.00	0.00	0.00	0.00	0.02	0.06	0.00
5	0.00	0.00	0.15	0.85	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.27	0.71	0.00	0.00	0.02	0.00	0.00
7	0.00	0.00	0.00	0.00	0.39	0.52	0.02	0.02	0.05	0.00
8	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.88
F	0.00	0.50	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00
W	0.50	0.07	0.07	0.14	0.07	0.07	0.00	0.07	0.00	0.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
G	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

P2 =

	3	4	5	6	7	8	F	W	D	G
3	0.46	0.29	0.00	0.00	0.00	0.00	0.23	0.02	0.00	0.00
4	0.00	0.36	0.51	0.00	0.00	0.00	0.09	0.02	0.02	0.00
5	0.00	0.00	0.27	0.57	0.00	0.00	0.16	0.00	0.00	0.00
6	0.00	0.00	0.00	0.33	0.53	0.00	0.14	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.19	0.65	0.11	0.00	0.05	0.00
8	0.00	0.00	0.00	0.00	0.00	0.38	0.07	0.00	0.00	0.55
F	0.79	0.06	0.06	0.09	0.00	0.00	0.00	0.00	0.00	0.00
W	0.53	0.07	0.07	0.20	0.00	0.13	0.00	0.00	0.00	0.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
G	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

P5 =

	3	4	5	6	7	8	F	W	D	G
3	0.47	0.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.40	0.52	0.00	0.00	0.00	0.02	0.01	0.04	0.00
5	0.00	0.00	0.26	0.69	0.00	0.00	0.03	0.00	0.03	0.00
6	0.00	0.00	0.00	0.31	0.67	0.00	0.00	0.02	0.00	0.00
7	0.00	0.00	0.00	0.00	0.50	0.44	0.00	0.00	0.06	0.00
8	0.00	0.00	0.00	0.00	0.00	0.35	0.00	0.00	0.04	0.62
F	0.50	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
W	0.50	0.36	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
G	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

P3 =

	3	4	5	6	7	8	F	W	D	G
3	0.57	0.42	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00
4	0.00	0.25	0.65	0.00	0.00	0.00	0.02	0.00	0.08	0.00
5	0.00	0.00	0.03	0.90	0.00	0.00	0.00	0.03	0.05	0.00
6	0.00	0.00	0.00	0.15	0.85	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.38	0.54	0.00	0.00	0.08	0.00
8	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.00	0.00	0.89
F	0.00	0.17	0.00	0.17	0.17	0.50	0.00	0.00	0.00	0.00
W	0.39	0.22	0.06	0.11	0.00	0.22	0.00	0.00	0.00	0.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
G	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

P6 =

	3	4	5	6	7	8	F	W	D	G
3	0.57	0.40	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00
4	0.00	0.53	0.43	0.00	0.00	0.00	0.00	0.01	0.03	0.00
5	0.00	0.00	0.25	0.69	0.00	0.00	0.04	0.00	0.02	0.00
6	0.00	0.00	0.00	0.41	0.51	0.00	0.03	0.05	0.00	0.00
7	0.00	0.00	0.00	0.00	0.40	0.57	0.00	0.00	0.04	0.00
8	0.00	0.00	0.00	0.00	0.00	0.45	0.00	0.00	0.00	0.55
F	0.60	0.20	0.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00
W	0.00	0.20	0.40	0.00	0.40	0.00	0.00	0.00	0.00	0.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
G	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Fig.2 PTM for each semester

P ave =

	3	4	5	6	7	8	F	W	D	G
3	0.53	0.42	0.00	0.00	0.00	0.00	0.04	0.01	0.00	0.00
4	0.00	0.39	0.52	0.00	0.00	0.00	0.02	0.01	0.05	0.00
5	0.00	0.00	0.17	0.75	0.00	0.00	0.04	0.01	0.03	0.00
6	0.00	0.00	0.00	0.30	0.65	0.00	0.03	0.02	0.00	0.00
7	0.00	0.00	0.00	0.00	0.36	0.55	0.02	0.00	0.06	0.00
8	0.00	0.00	0.00	0.00	0.00	0.28	0.01	0.00	0.01	0.71
F	0.40	0.25	0.03	0.09	0.04	0.09	0.10	0.00	0.00	0.00
W	0.36	0.19	0.13	0.10	0.11	0.11	0.00	0.01	0.00	0.00
D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
G	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Fig.3 Expected PTM

Table 2. Prob. of the students' progression to the next study stage during one academic semester

Fraction of students progressed to higher stage	3→4	4→5	5→6	6→7	7→8	8→G
Spring 2014/15	0.341	0.596	0.833	0.656	0.576	0.75
Spring 2016/17	0.286	0.509	0.568	0.528	0.649	0.552
Fall 2017/18	0.415	0.646	0.9	0.846	0.538	0.892
Spring 2017/18	0.538	0.439	0.848	0.711	0.523	0.88
Fall 2018/19	0.531	0.517	0.686	0.667	0.44	0.615
Spring 2018/19	0.402	0.434	0.691	0.514	0.566	0.545
EXPECTED	0.419	0.524	0.754	0.654	0.549	0.706

Table 3. The expected enrolment (to graduation or dismiss) from a particular study stage

Study Stage	μ_1	μ_2	μ_3	μ_4	μ_6	μ_7	μ Expected
3	8.7	18.8	7.8	8.8	9.6	11.1	9.3
4	5.8	15.7	5.5	7.0	7.7	8.9	7.2
5	4.9	13.8	4.7	5.6	6.3	7.2	6.0
6	4.2	10.6	3.8	4.4	5.0	5.7	4.8
7	2.7	6.7	2.6	2.9	3.3	3.4	3.1
8	1.3	3.7	1.1	1.1	1.5	1.8	1.5
F	7.7	18.6	3.5	1.0	9.6	10.6	6.9
W	6.3	15.6	6.2	7.8	8.7	7.0	7.5

Table 4: The expected duration of the study from the third semester to graduation

Acad. Semester	E_{3G}
Spring 2014/15	11.983
Spring 2016/17	21.489
Fall 2017/18	10.705
Spring 2017/18	11.638
Fall 2018/19	12.469
Spring 2018/19	13.816
EXPECTED IN LAST 6 SEMESTERS	12.37

E. The graduation -Dismissal probability

The Graduation – Dismissal probabilities are calculated as the absorption probabilities from a given transient state. For this purpose, the fundamental matrices, as well as submatrices R, gathered from the probability transition matrices were used. The results are presented in Table 5.

F. Predicting the future enrolment of students

Let we assume the initial state in the academic semester Spring 2018/19. Using the frequency data from Table 1 the initial vector $p^{(0)}$ is estimated:

$P^{(0)}$ =[0.200 0.203 0.132 0.146 0.114 0.122 0.008 0.014 0.014 0.049] If we want to predict the enrolment of students for the following three academic semesters then, we calculated the vectors $P^{(1)}$, $P^{(2)}$ and $P^{(3)}$ according to equation 7 and using the expected probability transition matrix P. Then, the calculated probabilities are transformed into the absolute number of students. In calculations, we assumed that every academic semester 34 new student are entered to the study programme. According to the average number of students who entered in the last two semesters of the study period. The results are given in Table 6.

Table 5: The graduation – dismissal probability from a particular study stage.

t	**F1**		**F2**		**F3**		**F4**		**F5**		**F6**		**F**	
	Spring 2014/2015		**Spring 2016/2017**		**Fall 2017/2018**		**Spring 2017/2018**		**Fall 2018/2019**		**Spring 2018/2019**		**Expected in Last 6 Semesters**	
	G	**D**	**G**	**D**	**G**	**D**	**G**	**D**	**G**	**D**	**G**	**D**	**G**	**D**
3	0.69	0.31	0.87	0.13	0.74	0.26	0.78	0.22	0.73	0.27	0.86	0.14	0.74	0.26
4	0.69	0.31	0.86	0.14	0.74	0.26	0.78	0.22	0.73	0.27	0.86	0.14	0.75	0.25
5	0.79	0.21	0.89	0.11	0.83	0.17	0.88	0.12	0.79	0.21	0.91	0.09	0.83	0.17
6	0.86	0.14	0.90	0.10	0.88	0.13	0.88	0.02	0.83	0.17	0.93	0.07	0.87	0.13
7	0.86	0.14	0.91	0.10	0.88	0.13	0.92	0.08	0.83	0.17	0.94	0.06	0.88	0.12
8	1.00	0.00	0.99	0.01	1.00	0.00	1.00	0.00	0.94	0.06	1.00	0.00	0.99	0.01
F	0.25	0.75	0.87	0.13	0.92	0.09	0.00	0.00	0.73	0.27	0.87	0.13	0.66	0.34
W	0.22	0.78	0.89	0.11	0.82	0.18	0.83	0.18	0.76	0.24	0.91	0.09	0.81	0.19

Table 6: Predicting of the future enrolment in the study programme.

YEAR	Semesters	3	4	5	6	7	8	F	W	D	G	∑
Fall 19/20	$P^{(1)}$	0.11	0.17	0.13	0.15	0.14	0.1	0.03	0.01	0.04	0.13	1
	NO. of Students	76	62	48	54	51	36	10	3	13	50	403
Spring 19/20	$P^{(2)}$	0.11	0.14	0.10	0.13	0.14	0.1	0.03	0.01	0.05	0.19	1
	NO.of Students	79	58	41	54	55	39	11	3	21	76	437
Fall 20/21	$P^{(3)}$	0.11	0.13	0.09	0.11	0.13	0.1	0.03	0.01	0.07	0.24	1
	NO. of Students	81	59	38	49	56	42	11	3	29	103	471

III. CONCLUSION

From a theoretical perspective, the study underscores the importance of using an absorbing Markov chain theory to study the pattern of students' enrolment and their academic performance within one of the engineering departments at engineering faculty in University of Benghazi. As for the study programme under consideration, the quantitative indicators calculated in previous sections provide some useful information to the programme manager. Results in Table 2 showed that in the average the fraction of students, who succeeded to moving from fifth to sixth semester was the best (75.4%) and then the fraction of students which succeeded to graduate (70.6%) (from eighth semester). Also, we can observe that the fraction of students which was succeeded to move to the next stage is decreased in the last two considered semesters. Furthermore, the findings of this study (as demonstrated in Table 2) indicate that the probability of progression from the third to the fourth semester is always lower than the probability of progression in the other semesters. This is quite an expected result since the experiences from the class show that the students become more serious and ambitious during their progression in the study. We can see from Table 3 that a student, who is currently enrolled in the third semester, needs on average 9.265 semesters to finish the study (graduation or dismissed). The results in Table 5 show that about 74.2% of these students will actually graduate, while the dismissed probability is about (25.8%).

Results in Table 5 indicated that the probability of graduation increases with increasing the level of study. furthermore, about 34% (on average) of students which froze their study programme will be dismissed and expected to leave the programme after 6.88 semesters (see table 3), while the expected duration of the study from the first semester in the department until graduation is 12.37 semesters (Table 4). Results in Table 4 also showed that the expected time to graduation is a rather constant and the values do not differ substantially during the semesters analyzed except in spring 2016/2017. Nevertheless, the numbers are quite high, considering the fact that the study programme lasts six semesters.

However, we can see from Table 5 that the probability of graduation increases with student’s progression over the study stages, and analogous, the probability of dismissed decreases. Such result was quite expected. It may indicate that when getting older, the students become more aware of their responsibilities and therefore become more successful with their study.

Finally, we can predict the future enrollment of students, it can be expected through $P^{(n)}$ which represents the fraction of students in a particular academic stage. Through the analysis, we observed that the number of students enrolled during the next three semesters will be increasing each semester. Thus, the analysis of students' performance using absorbing Markov chain provides a clear picture of education in the concerned department.

REFERENCES

- Adekele R., Oguntuase K., and Ogunsakin R., (2014), 'Application of Markov Chain to The Assessment of Students' Admission and Academic Performance in Ekiti State University', *International Journal of Scientific & Technology Research*, 3(7), pp. 349 – 357.
- Amiens E., Oisamoje M.,(2016) 'Students Flow in Private Universities in Nigeria: A Markov Chain Modelling Approach', European Journal of Business and Management, 8(8), pp. 107 – 114.
- Aparna, B., and Sarat, K., (2017), 'Stochastic process Approach to Analyse Students' Performance in Higher Education Institutions', *International Journal of Statistics and Systems*, 12(2), pp. 323 – 343.
- Auwalu A., Mohammed L., and Saliu A., (2013),'Application of Finite Markov Chain to a Model of Schooling', Journal of Education and Practice, 4(17),pp. 2 – 10.
- Brezavšček A., Bach M., and Baggia A., (2017), 'Markov Analysis of Students' Performance and Academic Progress in Higher Education' Organizacija, 50(2), pp. 83 – 95.
- Kemeny and Snell (1960), 'Finite Markov Chains' 2ed Edition, Princeton, NJ.
- Khairun Y., and Husna H.,(2011), 'Application of Markov chain in students' assessment and performance: a case study of School of Mathematical Sciences',*ICMSA 2020*, 36, pp. 2 -15.
- Nyandwaki et al (2014), 'Application of Markov Chain Model in Studying Progression of Secondary School Students by Sex During the Free Secondary Education: A Case Study of Kisi Central District', *Mathematical Theory and Modeling*, 4(4), pp. 73 – 84.
- Rahim R., et al., (2013),'Projection Model of Postgraduate Student Flow' *Applied Mathematics & Information Sciences*, 7(3), pp. 1 – 5.