Applications of First-Order Ordinary Differential Equations in Modeling Real-Life Phenomena

Asma Mustafa Abouethlah
Faculty of Science and Nature resources , Aljafara University, Libya
asma81farg@gmail.com

Siham Salih Khalleefah
Faculty of Science and Nature resources, Aljafara University, Libya

Abstract:

Mathematical modeling serves as a bridge between real-world phenomena and analytical understanding, often utilizing differential equations to describe the dynamics of various systems. First-order ordinary differential equations (ODEs), in particular, offer a powerful framework for capturing the behavior of systems governed by rate changes over time. This paper provides a systematic overview of the key applications of first-order ODEs in real-life scenarios, including population dynamics, Newton’s law of cooling, and radioactive decay. Through a qualitative examination of these models, the paper demonstrates how first-order ODEs can be effectively employed to describe, predict, and analyze natural and engineered systems. The study aims to highlight the theoretical and practical significance of first-order ODEs and encourage their broader application in scientific and engineering contexts.
Keywords: Applications of First-Order Ordinary differential equations, Population growth model, differential equations, Newton’s law of cooling.

الملخص:

تُشكّل النمذجة الرياضية جسرًا بين الظواهر الواقعية والفهم التحليلي، وغالبًا ما تستخدم المعادلات التفاضلية لوصف ديناميكيات الأنظمة المختلفة. تُقدّم المعادلات التفاضلية العادية من الدرجة الأولى (ODEs)، على وجه الخصوص، إطارًا فعّالًا لرصد سلوك الأنظمة التي تحكمها تغيرات المعدلات بمرور الوقت. تُقدّم هذه الورقة نظرة عامة منهجية على التطبيقات الرئيسية للمعادلات التفاضلية العادية من الدرجة الأولى في سيناريوهات الحياة الواقعية، بما في ذلك ديناميكيات السكان، وقانون نيوتن للتبريد، والانحلال الإشعاعي. من خلال دراسة نوعية لهذه النماذج، تُبيّن الورقة كيفية استخدام المعادلات التفاضلية العادية من الدرجة الأولى بفعالية لوصف الأنظمة الطبيعية والهندسية والتنبؤ بها وتحليلها. تهدف الدراسة إلى

تسليط الضوء على الأهمية النظرية والعملية للمعادلات التفاضلية العادية من الدرجة الأولى، وتشجيع تطبيقها على نطاق أوسع في السياقات العلمية والهندسية.

1. Introduction

First-order ordinary differential equations (ODEs) play a crucial role in modeling and understanding dynamic systems across various scientific and engineering fields. These equations describe how the state of a system evolves over time and provide a mathematical framework for analyzing phenomena ranging from population dynamics to chemical reactions, mechanical systems, and electrical circuits. The simplicity and versatility of first-order ODEs allow them to capture the essence of real-world problems, making them indispensable tools in both theoretical and applied research.

Many real-life problems in science and engineering, when formulated mathematically give rise to differential equation. In order to understand the physical behavior of the mathematical representation, it is necessary to have some knowledge about the mathematical character, properties and the solution of the governing differential equation (Lambe, and Tranter,. (2018)). Many of the principles, or laws, underlying the behavior of the natural world are statements or relations involving rates at which things happen. When it is expressed in mathematical terms, the relations are equations and the rates are derivatives (Logan, (2017)) . If we want to solve a real-life problem (usually of a physical nature), we first have to formulate the problem as a mathematical expression in terms of variables, functions, and equations. Such an expression is known as a mathematical model of the given problem. The process of setting up a model, solving it mathematically, and interpreting the result in physical or other term is called mathematical modeling (Bajpai et al. (2018).). Many physical problems in the fields of science, economics, engineering and technology remain meaningless without the application of differential equations to transform them into models. Frigon and Pouso (Frigon, and Pouso, (2017) studied the theory and applications of first-order ordinary differential equations which transformed the usual derivatives by Stielties derivatives. Rahan (Rehan, (2020)) investigated the first-order differential equation and Newton’s law of cooling. Some relevant works in the field of differential equations are found in (Simons, (2016), Ziv . (2013), Shior et al. (2022).). Tai-Ram (Hus (2018)). also studied applications of first-order ordinary differential equations in engineering analysis. This paper explores the practical significance of these equations through selected case studies. Real-world problems such as exponential population growth, Newton's law of cooling, and radioactive decay are modeled using first-order ODEs, illustrating how mathematical abstractions can be used to predict and control real phenomena. The main objective is to provide an accessible yet rigorous overview of how first-order ODEs are applied in real contexts, emphasizing both the simplicity and the power of these models.

2 .Preliminary Concepts

Definition 2.1 An equation containing the derivatives of one or more dependent variable, with respect to one or more independent variables, is said to be a differential equation (DE).

Definition 2.2 A differential equation is said to be an ordinary differential equation (ODE) if it contains only derivatives of one or more dependent variables with respect to a single independent variable. In symbols we can express an nth order ordinary differential equation in one dependent variable by the general form

F(x, y, y', y'' . . . .$y^{(n)}$) = o , (2.1)

where F denotes a mathematical expression involving $x, y, y', y'', y''', \ldots\ldots y^{(n-1)}, y^{n}$ and where

$$y^{(n)} = \frac{d^n y}{dx^n}$$

Definition 2.3 A partial differential equation is a differential equation which involves two or more variables and its partial derivative with respect to these variables.

Definition 2.4 The order of a differential equation is the order of the highest derivative in the equation.

Definition 2.5 The degree of a differential equation is the degree of the highest order derivative in the equation

Definition 2.6 First order first degree differential equation is a differential equation which contains no derivatives other than the first derivative and it has an equation of the form

$\frac{dy}{dx} = f(x, y)$, where y is a function of x (2.2)

and we rewrite this equation in the form $y' = \frac{dy}{dx} = f$(x,y)

Definition 2.7 An n^{th}-order ordinary differential equation is said to be linear in y if it can be written in the form

$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = f(x)$ (2.3)

where $a_0, a_1, a_2, \ldots, a_n$ and f are functions of x on some interval, and$a_n(x) \neq 0$. The functions $a_k(x)$, k= 0, 1, 2, ... , n are called the coefficient functions.

If n=1, in equation (2.3), we get a linear first order differential equation and it can be written in the form

$a_1(x) + a_0(x)y = f(x), a_1(x) \neq 0$ (2.4)

If $y' = \frac{dy}{dx}, \frac{a_0(x)}{a_1(x)} = p(x), \frac{f(x)}{a_1(x)} = q(x)$, equation (1.4) is equivalent to

$$\frac{dy}{dx} + p(x)y = q(x)$$

Definition 2.8 A differential equation that is not linear is called non-linear

3. Solution of First-Order ODEs

Here we discuss the solution of linear differential equation which divides in to two type one is of the form $y' + h(x)y = 0$ is known as homogeneous linear differential equation and its solution is $y = \frac{c}{u(x)}$

where is the constant of integration to be determined and $u(x)$ is known as the integrating factor can be

obtain from the equation. (Adkins, and Davidson: (2010)).

$u(x) = e^{\int -h(x)dx}$ (3.1)

If the first-order differential equation has the form $y' + h(x)y = g(x)$ is called

non homogeneous differential equation. The solution of this equation is similar with a little difference of the homogeneous differential equation

$y' + h(x)y = 0$

$y = \frac{1}{u(x)}(\int u(x).g(x)dx + a)$ (3.2)

where the function $u(x)$ can be obtain from Equation (3.1). (Hassan, and Zakari: (2018)) .

Example .

Obtain the solution of the initial value problem.

$y' - y.\tan x = \sin x$ where $y(0) = 1$.

Solution .

First we calculate integrating factor $u(x)$.

$$u(x) = e^{\int -h(x)dx} = e^{-\int \tan x dx}$$

Here

$$\int \tan x dx = \int \frac{\sin x}{\cos x}dx = \int \frac{-d(\cos x)}{\cos x} = -\int \frac{d(\cos x)}{\cos x} = -ln|\cos x|$$

Hence $u(x)$ is given by

$u(x) = e^{ln|\cos x|} = |\cos x|$.

We can take the function $u(x) = \cos x$ as the integrating factor make sour that the L.H.S of the equation is the derivative of the product$y(x)u(x)$.

Thus the general solution of the equation is

$$y(x) = \frac{1}{u(x)}\left[\int u(x)\sin x\, dx + c\right] = \frac{1}{\cos x}\left[\int \cos x \sin x\, dx + c\right] = \frac{c}{\cos x} - \frac{\cos 2x}{4\cos x}$$

Now we obtain the value of the constant c by putting $y(0) = 1$ in the above equation, thus we get

$$y(0) = \frac{c}{\cos 0} - \frac{\cos 0}{4\cos 0} \Longrightarrow 1 = c - \frac{1}{4} \Longrightarrow c = \frac{5}{4}$$

Hence the solution of the given problem is

$y(x) = \frac{5}{4\cos x} - \frac{\cos 2x}{4\cos x} = \frac{5-\cos 2x}{4\cos x}$ (Keryszig: (2006)).

4. Some Applications of First Order Differential Equation to Real Life Systems.

These are numerous real life applications of first-order differential equations to real life systems. In this study we shall discuss the following

- Population growth and decay
- Heat transferring
- Radioactive decay

4.1. Population Growth and Decay:

Population growth involves a dynamic process which can be developed using differential equations. The exponential growth model or natural growth model is known as Mathus' model (Jitender: (2022)). This model is based

on the assumption that the rate of change of the population is proportional to the existing population itself. If p(t) represents the total population at time t, the above assumption can be written as

$$\frac{dp}{dt} = kp(t) \qquad (4.1)$$

Where k is the proportionality constant .The above model (4.1) can also be used in the financial institute for example, when saving money in the bank, the balance in the savings account with interest compounded continuously exhibits natural growth provided no withdrawal and in this case the constant k represents the annual rate of interest ,group of animal populations grows exponential provided size is not affected by environmental factors, in this case, k is known as the productivity rate of population and it can also be used in migration.

Tractor factory the equation wit e^{-kt} ,the integrating factor $e^{-kt}\frac{dp}{dt} = kpe^{-kt}$

$$e^{-kt}\frac{dp}{dt} - kpe^{-kt} = 0\frac{dp}{dt}[pe^{-kt}] = 0$$

$\int\frac{dp}{dt}[pe^{-kt}] = \int 0$

$pe^{-kt} = C \; or \; p - ce^{kt}$

Suppose the initial population is p_0 then (0)= $p_o and\ c = p_o$

$(t) = p_o\, e^{kt}$ (4.2)

When k > 0 the population grows and when k < 0 ,the population decays

Example.

Suppose the population of a certain community is known to increase at a rate proportional to the number of people living in the community at time t ,the population has doubled after 7 years, how long would it take to triple? If it is known that the population of the community is 12,000 after 5 years ,determine the initial population and predict the population in 40 years.

Solution

Let p_0 denote the initial population of the community and (t) the population of the community at any time t, then from (4.1) we have

$\frac{dp}{dt} = kp \qquad p(t) = p_0\, e^{kt}$

From (3.2) given that

$(7) = p_0\, e^{7k} = 2$

$k = \frac{0.6931}{7} = 0.0990$

The solution of the model becomes

$(t) = p_0\, e^{0.0990t}$ (4.3)

Let t, be the time taken for the population to triple

then $3p_0 = p_0\, e^{0.0990t} \qquad e^{0.0990t} = 3$

0.0990t = ln3

$t\frac{1.0986}{0.0990} = 11.0970 \approx 11 years$

Applying (5) = 12,000

12,000 = $p_0\ e^{0.0990\times 5}$

$p_0 = \frac{12,000}{e^{0,450}}$ =7,315

Hence the initial population of the community was

$po \approx$ 7,315

Therefore, solution of the model is

(t) = $7315e^{0.0990t}$

So that the population in 40years is

(40) = $7315e^{40(0.0990)}$

(40) = $7315e^{3.960}$

(40) = 7315 (52,4573)

(40) ≈ 383,725

4.1.1. The logistic population model

Let (t) denotes the size of population of a country at any time t, then by Balance law for population, we have

$\frac{dp}{dt} = B(p,t) - D(p,t) + M(p,t)$ (4.4)

Where

B(p,t) represents inputs (birth rates), D(p,t) represents outputs (death rates), M(p,t) represents net migration.

One of the simplest cases is that assuming a model (4.4) for birth and death rates are proportional to the population and no migrants. Thus

$B(p,t)$=$bp(t)$, $D(p,t)$=$dp(t)$, $M(p,t)$=0

Hence equation (8) can be reduced to

$\frac{dp}{dt}$ =$(b-d)p$=γp, (4.5)

where $b-d$= γ is a proportionality constant which indicates population growth for γ>0 and population decay forγ <0, Since equation (4.5) is a linear differential equation, we can get a solution of the form:

$$\text{p}(t) = p_0 e^{rt}.$$

where $p(t_0) = p_0$ is the initial population and γ is called the growth or the decay constant. As a result, the population grows and continues to expand to infinity if

γ>0, while the population will shrink and tend to zero if $\gamma < 0$. However, populations cannot grow without bound there can be competition for food, resources or space. Suppose an environment is capable of sustaining no more than a fixed number $k$of individuals in its population. The quantity k is called the carrying capacity of the environment. Thus, for other models, equation (4.5) can be expected to decrease as the population p increases in size.

The assumption that the rate at which a population grows (or decreases) is dependent only on the number (t) present and not on any time-dependent mechanisms such as seasonal phenomena can be stated as

$$\frac{dp}{dt} = pf(p). \quad (4.6)$$

Now, assume that (p) is linear

$$f(p)= \alpha p+\beta$$

with conditions

$\lim_{p(t)\to 0} fp(t) = \gamma$, (k)=0 which leads $f(p) = \gamma-(\gamma/k)p$. Equation (10) becomes

$$\frac{dp}{dt} = p\left(\gamma - \frac{\gamma}{k}p\right) \quad (4.7)$$

This is called the logistic population model with growth rate γ and carrying capacity k. Clearly, when assuming p(t) is small compared to k, then the equation reduces to the exponential one which is nonlinear and separable. The constant solutions p=0 and p=k are known as equilibrium solution. From equation (4.7), we can have

$$(t)= \frac{kp_0}{p_0+(k-p_0)e^{-\gamma t}} \quad (4.8)$$

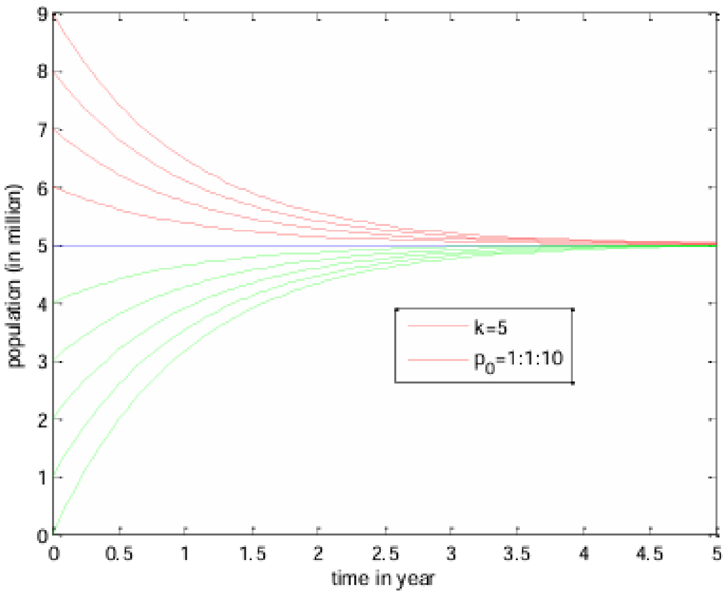

Figure 1: logistic population model with $\gamma = 1$

From Fig. 1, the following behaviors can be observed with the variation in the initial population as $t \rightarrow \infty$.

Value	Long term behavior of population	
$p_0 = 0$	$\lim_{t\to\infty} p(t) = 0$	no population
$0 < p_0 < k$	$\lim_{t\to\infty} p(t) = k$	population grows towards the balance population p = k
$p_0 = k$		population level or perfect balance with its surroundings
$p_0 > k$		population decreases towards the balance population p $= k$

4.2. Heat transferring:

Heat transferring is a process of transfer of heat from a body with higher temperature to a body with lower temperature. Hear the difference between the temperature is called potential for which transfer of heat is happen. There is different mode for heat transferring which are as follow

- Conduction
- Convection
- Radiation

1) **Conduction** The process by which the heat is transfer from hot end of an object to its cold end is called conduction. It is also known as thermal conduction or heat conduction. Basically, in solid heat is transferred by the process of conduction.

2)

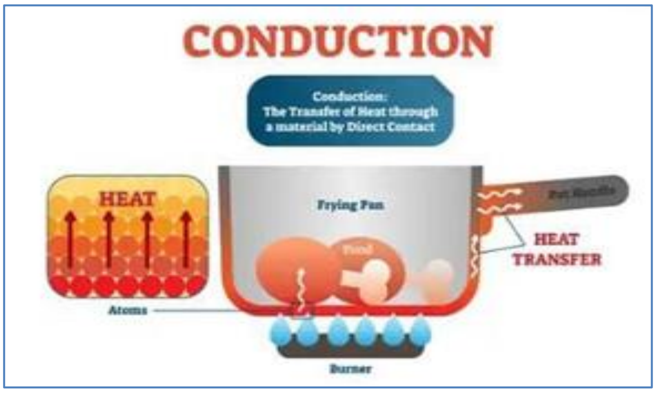

Figure 2

3) **Convection** The process by which fluid molecules moves from higher temperature region to lower temperature region is called convection.

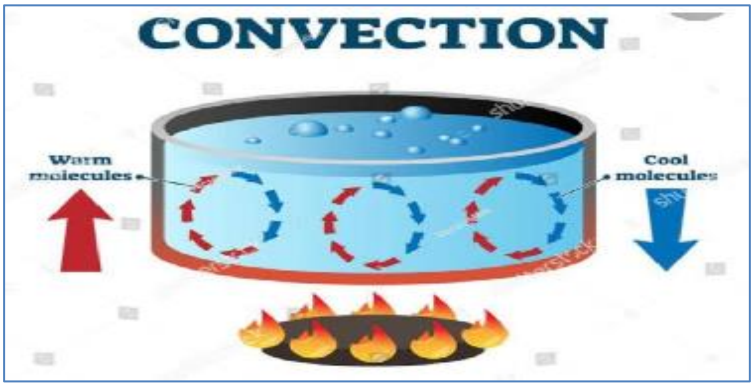

Figure 3

4) **Radiation** Radiation is the transfer of energy with the help of electromagnetic wave. It is generated by the emission of electromagnetic wave.

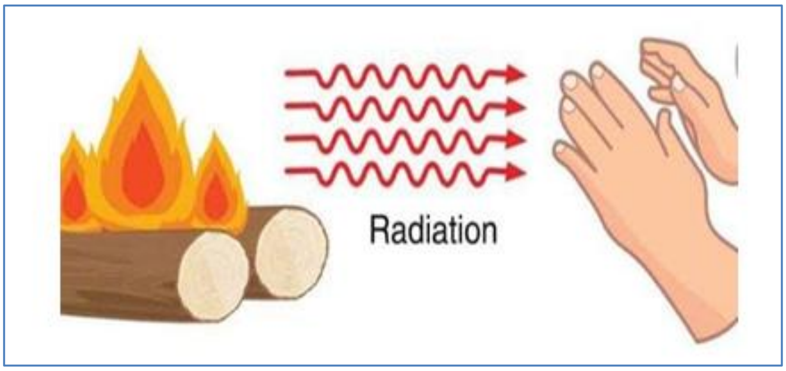

Figure 4

So above we have seen that heat flowing in solid by the process of conduction which we can determined by Fourier law. And we see in fluid, heat flowing by convection which we can determine by Newton's law of cooling.

4.3. Newton's Law of Cooling

Another important real-life application of differential equation is Newton's law of cooling. Sir Isaac Newton developed a huge interest in quantitative findings of the loss of heat in a body and a formula was derived to represent this phenomenon. The law states that the rate of change of temperature of a body is directly proportional to the difference between is solid object and the surrounding environment at a given instant of time.

$\frac{dT}{dt}\,\alpha(T_0 - T_5)$

$$\frac{dT}{dt} = k\,(T_0 - T_5) \qquad (4.9)$$

Where T(0) = T_0

T_0= Temperature of the body

T_5= Temperature of the surrounding

K = Constant of proportionality

$$\int \frac{dt}{T_0 - T_5} = \int Kdt$$

$$ln|T_0 - T_5| = kt + c$$

$$T_0 - T_5 = e^{kt+c}$$

Applying T(0) = T_0 yields

$$C = T_0 - T_5$$

$$T(t) = T_5 + (T_0 - T_5)e^{kt}$$

Suppose the temperatures at $t_1$and $t_2$are given we have

$$T(t_1) - T_5 = (T_0 - T_5)e^{kt_1}$$

$$T(t_2) - T_5 = (T_0 - T_5)e^{kt_2}$$

So that

$$\frac{T(t_1)-T_5}{T(t_1)-T_5} = e^{k(t_1-t_2)} \qquad (4.10)$$

Example 3 A police man discovered a dead body at midnight in a room where the temperature of the dead was 90^0F, the body temperature of the room was kept constant at 70^0F. Three hours later the temperature of the body dropped to 85^0F. Determine the time of death of the victim

Solution (0)= $98.6^0 F$ ($37^0 c$) = T_0 and T_5= $70^0 F$ Provided the victim was not sick

$$\frac{dT}{dt} = k(T_0 - 70) \ , \quad T(0) = 98.6$$

But

$$T(t) = T_5 + (T_0 - T_5)e^{kt}$$

So that

$$\frac{T(t_1) - T_5}{T(t_2) - T_5} = e^{k(t_1-t_2)}$$

$$T(t_1) = 90^0 f \qquad and \qquad T(t_2) = 85^0 f$$

$$\frac{90-70}{85-70} = e^{3k}$$

$$t_1 - t_2 = 3hours$$

$$k = \frac{1}{3}\ln\frac{4}{3} = 0.0959$$

$let\ t_1\ \ and\ \ t_2$ represent times of death and discovery of the dead body then

$$T(t_1) = T(0) = 98.6^0 F$$

$$And\ T(t_2) = 90^0 F$$

$$The\ time\ of\ death\ (t_3)\ =\ t_1 - t_2 and\ from\ (14)\ we$$

$$have$$

$$\frac{T(t_1) - T_5}{T(t_2) - T_5} = e^{kt_3}$$

$$\frac{89.6-70}{90-70} = e^{kt_3}$$

$$t_3 = \frac{1}{k}\ ln\frac{28.6}{20} \approx 3.730$$

Therefore, the person died at about 8:18 pm

4.4. RADIOACTIVE DECAY

In physics and chemistry, a radioactive element disintegrates when it emits energy in form of ionizing radiation. Substances that emit ionizing radiation are known as radionuclides. When a radioactive substance decays, a radionuclide transforms into different atom-a decay product. The atoms keep transforming to new decay products until a state is reached and are no longer radioactive. The radioactive law states that the probability per unit of time that a radioactive substance will decay is a constant and independent of time, which means that the number of nuclei undergoing decay per unit of time is proportional to the total number of nuclei in the given substance. (Harideo:(2013)).

The mathematical expression of the radioactive law is.

$$\frac{da}{dt} \propto A \qquad (4.11)$$

$$\frac{da}{dt} = KA$$

Where A(t) is the amount of substance and k is the constant of proportionality. Suppose the initial amount of the substance is Ao then A(0) = Ao and solving (4.11) using the initial condition we have

$$A(t) = A_0 e^{kt} \qquad (4.12)$$

Equation (4.12) is the solution of (4.11) where the constant k can be obtained from the half-life of the radioactive substance. The half-life of a radioactive material can be defined as the time it takes for one-half of the atom in an initial amount (Ao) to transform into atoms of the new element. Half-life determines the stability of a radioactive element. The half-life of a radioactive substance is directly proportional to its stability. Let T be the half-life of a radioactive element, then caphen $A(T) = \frac{A_0}{2}$ (4.13) Applying (4.12) and (4.13) we have,

$$\frac{A_0}{2} = A_0 e^{kt}$$

$$T = \frac{-ln}{k} \qquad (4.14)$$

Example

If the half-life of a radioactive element is 18days and we have 40g at the end of 30 days. Determine the amount of radioactive element present initially

Solution

Let A(t) represent the amount present at time t and Ao the initial amount of the element.

$$\frac{dA}{dt} = KA$$

A(40) = 30

Solving the IVP, yields.

$$A(t) = A_0 e^{kt}$$

But from (4.14)

$$K = \frac{-ln2}{T}$$

$$K = \frac{-ln2}{18} \qquad (4.15)$$

Applying A(40) = 30

$$40 = A_0 e^{30k}$$

$$A_0 = 40e^{-30k} \qquad (4.16)$$

Using (4.15) we have

$$A_0 = 40e^{\frac{30ln2}{18}}$$

$$A_0 = 127g$$

Conclusion:

First-order ordinary differential equations (ODEs) serve as a cornerstone in the mathematical modeling of dynamic systems, particularly those characterized by continuous change over time. This study has demonstrated the practical relevance and versatility of first-order ODEs through selected real-life applications such as population growth, Newton's law of cooling, and radioactive decay. These examples show how such equations can be effectively used to understand, predict, and analyze the behavior of various physical, biological, and engineering systems.

The physical growth and decay of any population which is well discussed in this article is of great concern to humanity this means that the .population growth model can be used to predict the population of a country in future when some facts about the country are known.

The logistic model remedies the weakness of exponential model. That is, the exponential model predicts either the population grows without bound or it decays to extinction.

But population cannot grow without bound as there can be competition for food, resources or space and this effect can be modeled by a logistic model by supposing that the growth rate depends on the population.

Newton's law of cooling states that the rate at which an objects cools is directly proportional to the difference in temperature between the object and its surrounding. It explains how fast an object is cool down. However, it works only if the difference in temperature between bodyand its surrounding must be small, the loss of heat from the body should be by radiation only. And the major limitation of newtons law of cooling is that the temperature of the surrounding must remain constant during the cooling of the body.

Radioactive decay is of great importance for the nucleus as the decay transforms it into a stable state, many of these modern technologies are products of radioactive decay, and a large amount of energy can be generated using decay in nuclear rector which is then converted to electrical energy for use in various form, in medical science, radioactive isotopes which can undergo radioactive has a great application because these isotopes are referred to as tracers and are injected into the body of a patient, in the body, the tracers gives off harmless radiation though may be detected through the device and this detection, scientists (physicians) can investigate blood flow to specific organs and evaluate organ function or bone growth. (Harideo:(2013)).

The reviewed models underscore the importance of qualitative and analytical approaches in solving first-order ODEs and interpreting their solutions in real-world contexts. By transforming abstract mathematical expressions into tools for decision-making and problem-solving, differential equations bridge the gap between theory and application.

Finally, this paper believed that many problems of future technologies will be solved using ordinary differential equations.

Future work can focus on expanding these models to include nonlinearities, external factors, or coupled systems involving higher-order or partial differential equations. Additionally, incorporating numerical simulation techniques and software tools such as MATLAB or Python can further enhance the analysis and provide more realistic solutions to complex systems.

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